Recitation cmps 211

Fall 2012

monday nov. 5

solved ex.

2.1 27, 33

2.2 13, 18, 29

2.3 3, 4, 36

4

Show $\phi \times A = A \times \phi$ 2.1 Q27

$$\Leftrightarrow \begin{cases} \phi \times A \subseteq \phi \\ A \times \phi \subseteq \phi \end{cases}$$

$$\Leftrightarrow \begin{cases} \forall x\,((x \in \phi \times A) \longrightarrow (x \in \phi)) \land \\ \forall x\,((x \in A \times \phi) \longrightarrow (x \in \phi)) \end{cases}$$

the premesis are universal statements, ~~therefore~~ Since ~~Use exhaustive prove~~ domain is infinite take random $x \in \phi \times A$

i) take random $x \in \phi \times A$

$= x \in \{(a,b) \mid (a \in \phi) \land (b \in A)\}$ by definition of power set

$= x \in \{(a,b) \mid \text{false} \land (b \in A)\}$

$= x \in \{(a,b) \mid \text{false}\}$

$= x \in \phi$

take random $x \in A \times \phi$

$= x \in \{(a,b) \mid (a \in A) \land (b \in \phi)\}$

$= x \in \{(a,b) \mid (a \in A) \land \text{false}\}$

$= x \in \{(a,b) \mid \text{falsle}\}$

$= x \in \phi$

$\phi \times A = A \times \phi = \phi$

3

2.1
Q33

$\exists x \in \mathbb{Z} (x^2 = 2)$

is equivelent in english to the square of some integer is 2

Proof existantial by construction. since disprooe

$\neg \exists x \in \mathbb{Z} (x^2 = 2)$ must be true

$\Longleftrightarrow \forall x \in \mathbb{Z}\ \neg(x^2 = 2)$

$\Longleftrightarrow \forall x \in \mathbb{Z}\ (x^2 \neq 2)$

universal proof since the Universe of discorse Which in this case is $\mathbb{Z}$ is infinite then Exhaustive Proof is not possible, proof by contradiction

assume $\exists x \in \mathbb{Z} (x^2 = 2)$

$\Longleftrightarrow x = \pm\sqrt{2}$ implying x is irrational

we reach a condruction of premise $x \in \mathbb{Z}$

hence $\forall x \in \mathbb{Z} (x^2 \neq 2)$ is true implying

$\exists x \in \mathbb{Z} (x^2 = 2)$ is false

4

~~one~~ reducing any integer by 1 is an integer

U.OD = set of integers (infinite)

$\forall x$ ~~$\in \mathbb{Z}$~~ $(x \in \mathbb{Z} \longrightarrow x-1 \in \mathbb{Z})$

2.1
34

Can't perform Exhaustive proof

Use contradiction to proof

$\forall x \, (x \in \mathbb{Z} \longrightarrow x-1 \in \mathbb{Z})$

$\Longleftrightarrow \forall x \, (\neg x \in \mathbb{Z} \lor x-1 \in \mathbb{Z})$

Proof by contradiction

assume $\neg \forall x \, (\neg x \in \mathbb{Z} \lor x-1 \in \mathbb{Z})$

$\Longleftrightarrow \exists x \, \neg(\neg x \in \mathbb{Z} \lor x-1 \in \mathbb{Z})$

$\Longleftrightarrow$ ~~$\exists x$~~ $(x \in \mathbb{Z} \land x-1 \notin \mathbb{Z})$

$\Longleftrightarrow$ ~~$\exists x$~~ (false)

$\therefore$ contradiction and $\forall x \, (x \in \mathbb{Z} \longrightarrow x-1 \in \mathbb{Z})$ is true

2.2 Q13

$A \cap (A \cup B) = A$

$$\begin{cases} A \cap (A \cup B) \subseteq A \\ A \subseteq A \cap (A \cup B) \end{cases}$$

$$\Leftrightarrow \begin{cases} \forall x \left((x \in A \cap (A \cup B)) \rightarrow (x \in A) \right) \wedge \\ \forall x \left((x \in A) \rightarrow (x \in A \cap (A \cup B)) \right) \end{cases}$$

(1) take random $x \in A \cap (A \cup B)$

$\Leftrightarrow x \in A \cap (A \cup B)$

$\Leftrightarrow x \in A \wedge ((x \in A) \vee (x \in B))$

$\Leftrightarrow ((x \in A) \wedge (x \in A)) \vee ((x \in A) \wedge (x \in B))$

$\Leftrightarrow x \in A$

$\therefore \forall x \left((x \in A \cap (A \cup B)) \rightarrow (x \in A) \right) \wedge$
$\forall x \left((x \in A) \rightarrow (x \in A \cap (A \cup B)) \right)$

$\therefore A \cap (A \cup B) = A$

2.2 18_a

$(A \cup B) \subseteq (A \cup B \cup C)$

Proof

$\forall x \left((x \in A \cup B) \rightarrow (x \in A \cup B \cup C) \right)$

take random $x \in A \cup B$

$\Leftrightarrow x \in A \cup B$

$\Leftrightarrow (x \in A) \vee (x \in B)$

$\Leftrightarrow (x \in A) \vee (x \in B) \vee (x \in C)$

$\Leftrightarrow (x \in A \cup B \cup C)$

$\therefore \forall x \left((x \in A \cup B) \rightarrow (x \in A \cup B \cup C) \right)$

$\therefore A \cup B \subseteq (A \cup B \cup C)$

Show $A - B = A \longrightarrow (A \cap B) = \phi$ $\boxed{\begin{matrix} 2.2 \\ Q29 \end{matrix}}$

Proof by contradiction

suppose $\{ (A \cap B) \neq \phi \ \wedge \ A - B = A \}$

$\Rightarrow \exists x \mid x \in (A \cap B)$

$\Leftrightarrow \quad x \in A \quad \wedge \ x \in B$

But $(A - B) = A$

$\Leftrightarrow \left(x \in (A-B) \quad \wedge \ (x \in B) \right)$

$\Leftrightarrow \ ((x \in A) \wedge \neg(x \in B)) \wedge (x \in B)$

$\Leftrightarrow \quad (x \in A) \wedge \left(\neg(x \in B) \wedge (x \in B) \right)$

$\Leftrightarrow \quad (x \in A) \wedge$ False

$\Leftrightarrow$ False

$\therefore$ Contradiction, hence $A - B = A$ implies that the intersection of ~~A∩B~~ A and B is ϕ

Section 2.3

~~[illegible]~~

Q3

2.3
3a

ⓐ is $f \; S \to Z$?

f(s) is the position of a 0 bit in S
the position of a 0 in a string returns the index of
0 which is for sure an integer because you can't
index with non-integers. for example
let $x \in S$
$x = 0110101$
~~which ind.~~ the index of which zero in x is returned?
the first? second or third occurance? Since ~~this~~
~~th~~ which occurance of zero is not defined then
the above is not a function from $f: S \to Z$

ⓑ f(s) is the number of 1 bits in S
the count of 1 bits in S is an integer as the
count will start from 0 and will be incremented
by 1 every time a 1 bit is found in S
for example let $x \in S$
$X = 0110101$
count of 1's $=$ ~~3~~ 4
therefore $f: S \to Z$ a function from the set
of all bit strings to the set of integers

find range and domain

2.3 Q4

Q4

(a) function that assigns to each non-negative integer its last digit

range of a function is the set of all images of elements in the domain

domain of a function is the set of elements on which the function is defined

in this case by the definition of ~~range~~ domain the ~~range~~ domain of this function is the set of non-negative integerts

$\mathbb{Z} = \{x \mid x \text{ is a non negative}\}$

$= \{0, 1, 2 \ldots\}$ the ~~range~~ domain is non-finite set

while by definition of ~~domain~~ range, the domain of this function is the last digit of a number.

a single ~~the~~ digit could only be between 0 -- 9 hence the domain

$D = \{x \mid x = 0, 1, 2 \ldots 9\}$

To Show $\underbrace{f(S \cup T)}_{\text{set of images of elements in } S\cup T} = f(S) \cup f(T)$

$\boxed{\begin{matrix} 2.3 \\ 36 \end{matrix}}$

$\Leftrightarrow \begin{cases} f(S \cup T) \subseteq f(S) \cup f(T) \\ f(S) \cup f(T) \subseteq f(S \cup T) \end{cases}$

$\Leftrightarrow \left\{ \begin{matrix} \forall x \left((x \in f(S \cup T)) \rightarrow (x \in f(S) \cup f(T)) \right) \wedge \\ \forall x \left((x \in f(S) \cup f(T)) \rightarrow (x \in f(S \cup T)) \right) \end{matrix} \right\}$

(1) take random $x \in f(S \cup T)$

$\Leftrightarrow \exists x' : ((x' \in S \cup T) \wedge f(x') = x$

$\Leftrightarrow (x' \in S) \vee (x' \in T) \wedge (f(x') = x$

$\Leftrightarrow ((x' \in S) \wedge f(x') = x) \vee ((x' \in T) \wedge f(x') = x)$ by distributivity

$\Leftrightarrow (x \in f(S)) \vee ((x \in f(T))$

$\Leftrightarrow x \in (f(S) \cup f(T))$